University of Colombo, Sri Lanka

University of Colombo School of Computing

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2023 — **3**rd Year Examination — Semester 5

IT5506 — Mathematics for Computing II

Structured Question Paper
(2 Hours)

To be completed by the candidate

Index Number							

Important Instructions

- The duration of the paper is **2 hours**.
- The medium of instructions and questions is English. Students should answer in the medium of English language only.
- This paper has **4 questions** on **14** pages. All questions carry 25 marks. Answer **ALL** questions.
- Write your answers **only on the space provided** on this question paper.
- Do not tear off any part of this question paper. Under no circumstances may this paper (or any part of this paper), used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper. If a page or part of a page is not printed, please inform the supervisor/invigilator immediately.
- Any electronic device capable of storing and retrieving text, including electronic dictionaries, smartwatches, and mobile phones, is not allowed.
- Calculators are **not allowed**.
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To be completed by the examiners

1	
2	
3	
4	
Total	

1.

(a) Given that square matrix S is denoted as $S = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

(i) Show that S^{-1} is exist.

(03 Marks)

$$\det(S) = 1 \cdot ((1)(2) - (1)(1)) - 1 \cdot ((2)(2) - (1)(1)) + 1 \cdot ((2)(1) - (1)(1))$$

Simplify the terms inside the parentheses:

$$\det(S) = 1 \cdot (2 - 1) - 1 \cdot (4 - 1) + 1 \cdot (2 - 1)$$

$$\det(S) = 1 \cdot 1 - 1 \cdot 3 + 1 \cdot 1$$

$$\det(S) = 1 - 3 + 1 = -1$$

Det(S) is non-zero.

(ii) Find the inverse of the matrix S by using the Gauss-Jordan method.

(07 Marks)

Start with the augmented matrix [S|I]

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

…… ….. After row operations ends with

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 3 & -1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$S^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 3 & -1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

(b) Matrix multiplication is not commutative in general, but there are instances where it satisfies the commutative property

(i) Show that for two matrices A and B, it is not always true that AB=BA

(03 Marks)

Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

1. Compute AB:

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

2. Compute BA:

$$BA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

Clearly, $AB \neq BA$, demonstrating that matrix multiplication is not commutative.

Or any other counter-example

(ii) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ then show that $AB = BA = 8I_3$

(04 Marks)

(iii) If $AB = BA = 8I_3$ then show that B^{-1} =(1/8)A

(02 Marks)

1. Compute AB.
2. Compute BA.

- $(AB)_{11} = (-4)(1) + (4)(1) + (4)(2) = -4 + 4 + 8 = 8$
- $(AB)_{12} = (-4)(-1) + (4)(-2) + (4)(1) = 4 - 8 + 4 = 0$
- $(AB)_{13} = (-4)(1) + (4)(-2) + (4)(3) = -4 - 8 + 12 = 0$

Second row of AB:

- $(AB)_{21} = (-7)(1) + (1)(1) + (3)(2) = -7 + 1 + 6 = 0$
- $(AB)_{22} = (-7)(-1) + (1)(-2) + (3)(1) = 7 - 2 + 3 = 8$
- $(AB)_{23} = (-7)(1) + (1)(-2) + (3)(3) = -7 - 2 + 9 = 0$

Third row of AB:

- $(AB)_{31} = (5)(1) + (-3)(1) + (-1)(2) = 5 - 3 - 2 = 0$
- $(AB)_{32} = (5)(-1) + (-3)(-2) + (-1)(1) = -5 + 6 - 1 = 0$
- $(AB)_{33} = (5)(1) + (-3)(-2) + (-1)(3) = 5 + 6 - 3 = 8$

So, the matrix AB is:

$$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

Similarly calculate BA

3. Verify if both results yield $8I_3$, where I_3 is the 3×3 identity matrix and $8I_3 = 8 \times I_3$.

(iv) Represent the following system of linear equations in matrix form $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$. Using the results from part (a)(ii) and (a)(iii), solve the system of equations.

(06 Marks)

03 Marks for Representing the system of linear equations in matrix form

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

Full marks (03) are awarded for solving the system of equations. The question needs to be corrected to: 'Using the results from part (b)(ii) and (b)(iii), solve the system of equations

Invers of $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ is (1/8) $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ refer (a)(iii) and (a)(ii)

Hence

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$$\frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

The solution is ($x = 3, y = -2, z = -1$).

2.

(a). Let V be a vector space over the field F, and let $S \subseteq V$ be a subset of V

(i) For S to be a subspace over V, what conditions must be satisfied?

(Marks 06)

For a subset S of a vector space V over a field F to be a **subspace** of V, it must satisfy three essential conditions:

1. **Contains the Zero Vector**: The zero vector $\mathbf{0} \in V$ must be in S. In other words, $\mathbf{0} \in S$.
2. **Closed under Addition:** If $\mathbf{u}, \mathbf{v} \in S$, then the sum $\mathbf{u} + \mathbf{v}$ must also be in S. This means:

$$\mathbf{u}, \mathbf{v} \in S \quad \Rightarrow \quad \mathbf{u} + \mathbf{v} \in S.$$

3. **Closed under Scalar Multiplication**: If $\mathbf{v} \in S$ and $c \in F$ (a scalar), then the scalar multiple $c\mathbf{v}$ must also be in S. In other words:

$$\mathbf{v} \in S \quad \text{and} \quad c \in F \quad \Rightarrow \quad c\mathbf{v} \in S.$$

(ii) Any line through the origin is given by $S = \{ \begin{bmatrix} x \\ y \end{bmatrix} : ax + by = 0\,, a, b \in \mathbb{R}^2 \}$. Is S a subspace of $\mathbb{R}^2$? Justify your answer.

(Marks 06)

1. **Contains the Zero Vector**:

The zero vector in $\mathbb{R}^2$ is $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Substituting $x = 0$ and $y = 0$ into the equation $ax + by = 0$ gives:

$$a(0) + b(0) = 0,$$

which is true. Therefore, the zero vector is in S.

2. **Closed under Addition:**

Let $\mathbf{u} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ be two vectors in S, so that $ax_1 + by_1 = 0$ and $ax_2 + by_2 = 0$.
We need to check if their sum $\mathbf{u} + \mathbf{v} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$ is also in S, i.e., if it satisfies the equation $ax + by = 0$:

$$a(x_1 + x_2) + b(y_1 + y_2) = (ax_1 + by_1) + (ax_2 + by_2) = 0 + 0 = 0.$$

Therefore, $\mathbf{u} + \mathbf{v} \in S$, so S is closed under addition.

3. **Closed under Scalar Multiplication**:

Let $\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix} \in S$, so that $ax + by = 0$. For any scalar $c \in \mathbb{R}$, we need to check if $c\mathbf{v} = \begin{pmatrix} cx \\ cy \end{pmatrix}$ satisfies the equation $ax + by = 0$. Substituting:

$$a(cx) + b(cy) = c(ax + by) = c(0) = 0.$$

Therefore, $c\mathbf{v} \in S$, so S is closed under scalar multiplication.

S satisfies all three conditions — it contains the zero vector, is closed under addition, and is closed under scalar multiplication S **is a subspace**

(b). Determine whether the following function T is a *linear transformation*.

$$T: \mathrm{R}^3 \rightarrow \mathrm{R}^3, \text{ with } T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x^3 \\ y^2 \\ \sin z \end{bmatrix}$$

(Marks 06)

Let $\mathbf{u} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$. We need to check if:

$$T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$$

First, calculate $T(\mathbf{u} + \mathbf{v})$:

$$\mathbf{u} + \mathbf{v} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{pmatrix}$$

Now apply T:

$$T(\mathbf{u} + \mathbf{v}) = \begin{pmatrix} (x_1 + x_2)^3 \\ (y_1 + y_2)^2 \\ \sin(z_1 + z_2) \end{pmatrix}$$

Now calculate $T(\mathbf{u}) + T(\mathbf{v})$:

$$T(\mathbf{u}) = \begin{pmatrix} x_1^3 \\ y_1^2 \\ \sin(z_1) \end{pmatrix}, \quad T(\mathbf{v}) = \begin{pmatrix} x_2^3 \\ y_2^2 \\ \sin(z_2) \end{pmatrix}$$

$$T(\mathbf{u}) + T(\mathbf{v}) = \begin{pmatrix} x_1^3 \\ y_1^2 \\ \sin(z_1) \end{pmatrix} + \begin{pmatrix} x_2^3 \\ y_2^2 \\ \sin(z_2) \end{pmatrix} = \begin{pmatrix} x_1^3 + x_2^3 \\ y_1^2 + y_2^2 \\ \sin(z_1) + \sin(z_2) \end{pmatrix}$$

Clearly, $T(\mathbf{u} + \mathbf{v}) \neq T(\mathbf{u}) + T(\mathbf{v})$ in general because:

$$(x_1 + x_2)^3 \neq x_1^3 + x_2^3 \quad \text{and} \quad (y_1 + y_2)^2 \neq y_1^2 + y_2^2 \quad \text{and} \quad \sin(z_1 + z_2) \neq \sin(z_1) + \sin(z_2)$$

Thus, **additivity does not hold**.

Hence T is not a *linear transformation*.

(c). Consider given tow vector s $\vec{A} = [2 \quad 3 \quad 4]$ and $\vec{B} = [1 \quad 0 \quad -1]$ in R^3

(i) Calculate the magnitudes (lengths) of $\vec{A}$ and $\vec{B}$

(02 Marks)

The magnitude of a vector $\vec{V} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ is given by:

$$|\vec{V}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

For $\vec{A} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$:

$$|\vec{A}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$$

For $\vec{B} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$:

$$|\vec{B}| = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{1 + 0 + 1} = \sqrt{2}$$

(ii) Find the unit vectors along $\vec{A}$ and $\vec{B}$

(02 Marks)

Unit vector along $\vec{A}$:

$$\hat{A} = \frac{1}{|\vec{A}|} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

Unit vector along $\vec{B}$:

$$\hat{B} = \frac{1}{|\vec{B}|} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

(iii) Compute the angle between vectors $\vec{A}$ and $\vec{B}$

(03 Marks)

The angle θ between two vectors $\vec{A}$ and $\vec{B}$ can be found using the dot product formula:

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

Rearranging for $\cos\theta$:

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$$

Step 1: Find the dot product $\vec{A} \cdot \vec{B}$

$$\vec{A} \cdot \vec{B} = (2)(1) + (3)(0) + (4)(-1) = 2 + 0 - 4 = -2$$

$$|\vec{A}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$$

$$|\vec{B}| = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{1 + 0 + 1} = \sqrt{2}$$

Get cos invers to find θ

3)

(a) Write down two (2) key differences between integer programming and linear programming?

Linear Programming (LP) allows decision variables to take any real values within the given constraints.
Integer Programming (IP) requires some or all decision variables to be integers, which can be either binary or general integers.

LP problems can be solved efficiently using the Simplex method.
IP problems are generally NP-hard and require more complex algorithms such as Branch and Bound to find optimal solutions.

Or Similar Answers

(b) Explain basic components of a linear programming model, providing examples for each component.

(06 Marks)

Decision Variables – The variables that represent the choices available in the optimization problem.

Objective Function – A linear function that needs to be maximized or minimized (e.g., profit maximization or cost minimization).

Constraints – A set of linear inequalities or equations that define the feasible region by restricting the values of decision variables.

Non-Negativity Restriction – Decision variables must be zero or positive (i.e.,

Feasible Region – The set of all possible solutions that satisfy the constraints.

Optimal Solution – The point within the feasible region that gives the best (maximum or minimum) value of the objective function.

(c) A company, XYZ, manufactures two (2) types of cars: Sedans and SUVs. Assembling a Sedan requires three (3) hours, while an SUV requires four (4) hours. The total time available for assembling cars is limited to 60 hours. Additionally, the company aims to produce at least twice as many Sedans as SUVs to meet sales goals. Each Sedan generates a profit of $200, and each SUV generates a profit of $100.

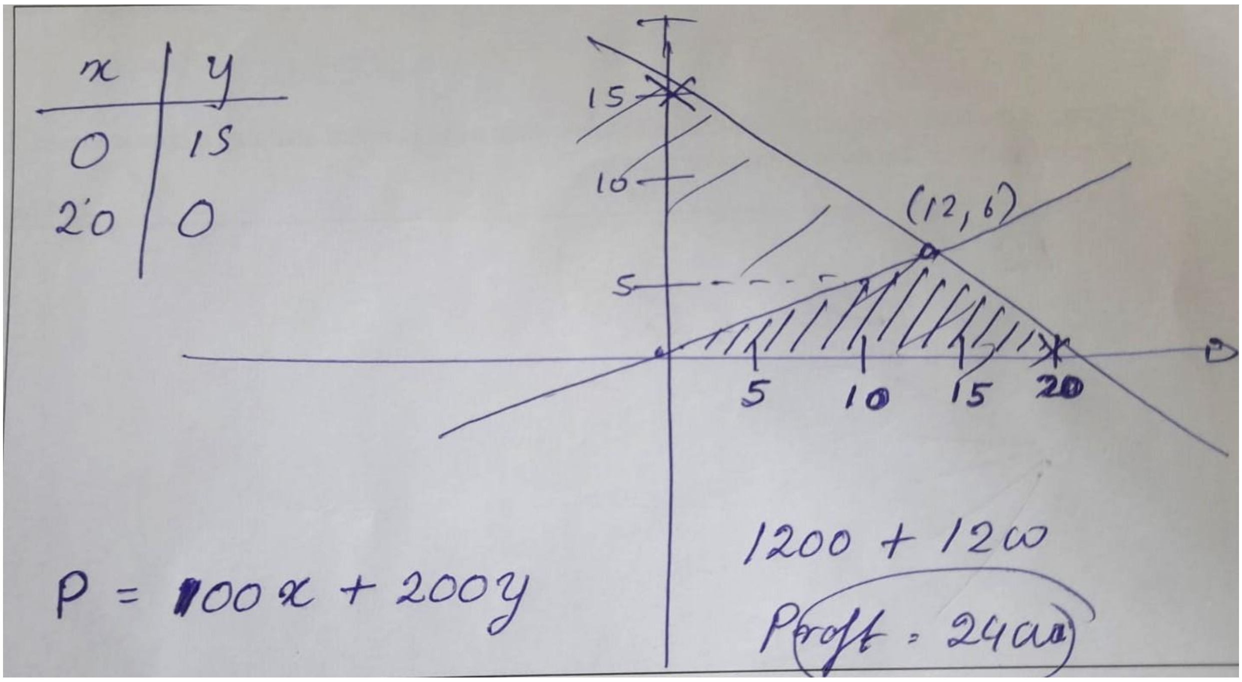
(i) Formulate a linear programming problem to determine the number of each type of car the company should produce to maximize its profit.

X = # of Sedan Cars Produced
Y = # of SUV cars produced

3X + 4Y <= 60
X >= 2Y
X >= 0
Y >=0

Z = 200X + 100Y

(ii) Use the graphical method or any other method to solve the problem and find the number of each type of car the company should produce to maximize its profit.



(4)
(a) Determine the total number of possible outcomes when a die is rolled and then a coin is tossed. List all the outcomes in the form of ordered pairs.

Step 1:
Rolling a Dice
{1,2,3,4,5,6}

Tossing a coin:
{H,T}

Step 2:
Calculate the Total Number of Outcomes
$6 \times 2 = 12$

Step 3: List All Possible Outcomes as Ordered Pairs
(1,H), (1,T), (2,H), (2,T), (3,H), (3,T), (4,H), (4,T), (5,H), (5,T), (6,H), (6,T)

(b)

How many different license plates can be created if a license plate consists of two letters followed by four digits, and the second letter cannot be “O”?

$$\text{Total Plates} = (\text{Choices for 1st letter}) \times (\text{Choices for 2nd letter}) \times (\text{Choices for 4 digits})$$

$$= 26 \times 25 \times 10,000$$

$$= 26 \times 25 \times 10^4$$

$$= 650 \times 10,000$$

$$= 6,500,000$$

(c)

A six-sided die is rolled twice. Let X represent the sum of the numbers obtained on the two rolls.

Find: (i) P(X=7) (ii) P(X>8)

(i)

We count the number of outcomes where the sum is 7:

$$(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)$$

There are **6** such outcomes. The probability is:

$$P(X=7) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{6}{36} = \frac{1}{6}$$

(ii)

$$P(X > 8) = \frac{10}{36} = \frac{5}{18}$$

(d) A factory produces a batch of 5 electronic devices, and each device has a 0.8 probability of passing quality control. What is the probability that at least 3 devices pass quality control?

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$$

Case: $X = 3$

$$P(X = 3) = \binom{5}{3}(0.8)^3(0.2)^2$$

Case: $X = 4$

$$P(X = 4) = \binom{5}{4}(0.8)^4(0.2)^1$$

Case X = 5

$$P(X = 5) = \binom{5}{5}(0.8)^5(0.2)^0$$

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$$

$$= 0.2048 + 0.4096 + 0.32768$$

$$= 0.94208$$

(e)

A theme park manager observes that guests arrive at the entrance gate at an average rate of 2 guests per minute during the peak hours of 10.00 a.m. to 11.00 a.m. Using this information, determine the following:

(i) Exactly 2 guests arrive between 10:57 a.m. and 11:00 a.m.

(ii) Fewer than 4 guests arrive between 10:57 a.m. and 11:00 a.m.

The Poisson probability formula is:

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

$$\lambda = 2 \times 3 = 6$$

(i)

$$P(X = 2) = \frac{e^{-6}6^2}{2!}$$

$$= \frac{e^{-6} \times 36}{2}$$

$$= \frac{36e^{-6}}{2}$$

$$= 18e^{-6}$$

(ii)

$$P(X < 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

$$P(X < 4) = 0.002478 + 0.0149 + 0.0446 + 0.0892$$

$$= 0.1512$$